## The Interference between Continuum and Resonance in $e^+e^- \rightarrow c\bar{c}$ Experiment

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 $e^+e^-$  experiments at charmonium production threshold are reviewed, it is found that the contribution of the continuum process via virtual photon has been neglected in almost all the experiments and most channels analyzed. It is pointed out that the contribution of the continuum part may affect the final results significantly in  $\psi(2S)$  and  $\psi(3770)$  decays, while the interference between the continuum amplitude and the resonance amplitude may even affect the  $J/\psi$  decays as well as the  $\psi(2S)$  and  $\psi(3770)$ . This leads to the revise of the analysis of strong and electromagnetic amplitude in  $\psi(2S)$  decays, including  $\psi(2S) \to VP$  which is the long lasting puzzle between  $J/\psi$  and  $\psi(2S)$  decays. For  $\psi(3770)$  physics, a large constructive interference for light hadron modes and destructive interference for  $D\overline{D}$  could be responsible for the discrepancy between the larger cross section of inclusive hadrons by direct measurement of  $e^+e^- \to \psi(3770) \to hadrons$  than the  $D\overline{D}$  cross section measured using D single-tag and double-tag method.

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#### I. INTRODUCTION

It is well known that the  $e^+e^-$  experiments have lots of advantages in particle physics study: large cross section, small background, and well-determined initial state (both four-momentum and quantum numbers). There were lots of  $e^+e^-$  experiments, there are still many  $e^+e^$ experiments working, and there will be further experiments build to continue the experimental study. With the energy ranges from  $\pi^+\pi^-$  threshold up to TeV scale, these experiments contribute a lot to the knowledge of the world around us. Among them, there are some working at the  $\tau$ -charm energy region, where the  $J^{PC} = 1^{--}$  charmonium states are produced and studied, including Mark-I, Mark-II, Mark-III, DM2, Crystal Ball, BES and so on. Recently, CLEO working at CESR decided lower its energy from the  $B\overline{B}$  threshold to the charm threshold [1], and BES working at BEPC decided upgrade both the accelerator and the detector to make a factory-like experiment [2], these two experiments will reach an extremely high precision in the study of the charmed mesons and the dynamics of the charmonium states decay in this energy range.

 $J/\psi$ , the first vector charmonium state discovered in 1974 [3], gains lots of attention due to its surprising narrow width and strong coupling to  $e^+e^-$  state. Since then, it has been used as an ideal laboratory for light hadron spectroscopy and charmonium decay dynamics study, which are the essential tasks of the low energy QCD. The attempt of figuring out the strong decays of  $J/\psi$  via three-gluon and electromagnetic decays via one-photon annihilation reveals the relative phase between these two amplitudes may be large, with the help of some pure electromagnetic decay modes of  $J/\psi$  like  $\omega \pi^0$  [4].

This is an important information since it indicates there would be no interference between these two amplitudes. This situation will be further studied in this paper.

 $\psi(2S)$ , the radially excited spin triplet state of  $J/\psi$ , has also the narrow nature and strong coupling to  $e^+e^-$  state, but most impressing feature found in the  $\psi(2S)$  study is the abnormal suppression of some decay modes compared with the corresponding  $J/\psi$  decays based on perturbative QCD predictions. This suppression was first observed by the Mark-II experiment in vector pseudoscalar (VP) decay modes like  $\rho\pi$  and  $K^*\overline{K}$  [5], and confirmed by BES [6] (referred as " $\rho\pi$  puzzle" in literatures). Moreover, BES also observed the suppression in vector tensor (VT) decays of  $\psi(2S)$  [7]. This has led to substantial theoretical efforts in solving the problem [8, 9], unfortunately, most of the models were ruled out by the experiments, while some others need further experimental data to test.

The  $\psi(3770)$ , currently regarded as the D wave charmonium state, lies above the  $D\overline{D}$  threshold, as a consequence, the OZI allowed decays of  $\psi(3770) \to D\overline{D}$  would dominate its decays. This picture has been considered true for pretty long period of time, until recently, a careful study of the old analyses indicates the  $D\overline{D}$  cross section may be lower than inclusive hadron cross section of  $\psi(3770)$ , or in other words, there are substantial non- $D\overline{D}$  decays of  $\psi(3770)$  state [10].

The three topics in charmonium studies (relative phase between strong and electromagnetic decays, " $\rho\pi$  puzzle", and non- $D\overline{D}$  decays of  $\psi(3770)$ ) play important roles in understanding the charmonium decay dynamics. In following sections, we will examine carefully the experimental observables and theoretical expectations in charmonium study in  $e^+e^-$  experiments, to provide a possibility of investigating these problems in a self-consistent way by considering the unavoidable background process in  $e^+e^-$  experiment, namely, the continuum process. We argue that, for any exclusive decay final states of these charmo-

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nia decay, in some cases, the contribution of this process to the amplitude may be very important, while in some other cases, although the direct contribution is relatively small, the interference between this term and other dominant amplitudes may contribute a non-negligible part, which maybe provide a guideline to solve the three existing problems in the charmonium decays.

The purpose of our paper focuses on locating the natural source of the existing problems rather than to offer a detailed solutions of them. So we begin with the inclusive hadronic process to exhibit the experimental effect on theoretical cross section, then simple assumption is often adopted to estimate the function on exclusive processes from the continuum contribution. At the last of the paper, we also studied the experimental condition dependence of the results in case the interference was not considered, which is true for most of the existing experimental results on  $J/\psi$ ,  $\psi(2S)$  and  $\psi(3770)$  decays.

# II. EXPERIMENTALLY OBSERVED CROSS SECTIONS

We know that  $J/\psi$  and  $\psi(2S)$  decay into light hadrons through two interactions: the three-gluon strong interaction and the one-photon electromagnetic interaction. There is in general a relative phase between these two amplitudes. This is also true for  $\psi(3770)$  in its OZI suppressed decay into light hadrons. These two amplitudes and the phase between them are extracted from experimental data by several authors for  $J/\psi$  and  $\psi(2S)$  [4, 9, 11]. If we denote the amplitude of three-gluon by  $a_{3g}$  and one-photon by  $a_{\gamma}$ , both of them can be complex, the decay rate

$$\sigma \propto |a_{3q} + a_{\gamma}|^2 \quad . \tag{1}$$

In  $e^+e^-$  colliding beam experiments, the charmonium are produced from  $e^+e^-$  annihilation, there is inevitable another amplitude

$$e^+e^- \to \gamma^* \to hadrons$$
 (2)

accompanied with the production of the resonances. This amplitude does not go through the resonance, but in general it could produce the same final hadronic states as charmonium decays do. So there are three Feynman diagrams corresponding to the experimentally measured cross sections, i.e. the three-gluon decays, the one-photon decays, and the one-photon continuum process, as illustrated in Fig. 1. The former two amplitudes are associated with the resonance, while the last one is a slowly varying function of C.M. energy  $(\sqrt{s})$ . To analyze the experimental results, we must take into account three amplitudes and two phases. Taking the amplitude of one-photon continuum as  $a_c$ , the experimentally observed cross section

$$\sigma' \propto |a_{3g} + a_{\gamma} + a_c|^2 \quad . \tag{3}$$

In this paper, for simplicity, we define

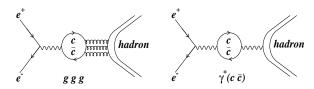
$$a_c = \frac{e^2 \mathcal{F}(s)}{s} e^{i\phi'} \quad , \tag{4}$$

where e is the electromagnetic coupling constant,  $\mathcal{F}(s)$  is the form factor, and

$$a_{3g} = \frac{\mathcal{C}_{ggg}}{s - M^2 + iM\Gamma} \quad , \tag{5}$$

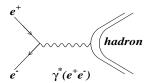
$$a_{\gamma} = \frac{\mathcal{C}_{\gamma} e^{i\phi}}{s - M^2 + iM\Gamma} \quad , \tag{6}$$

where  $C_{ggg}$  and  $C_{\gamma}$  are taken to be real, and M and  $\Gamma$  are the mass and the width of the resonance. We shall use this form for general discussions and numerical calculations.



(a) three gluon process

(b) one photon process



(c) one photon continuum process

FIG. 1: Three Feynman diagrams of hadron process.

The experimentally observed cross sections in  $e^+e^-$  collision are modified by the initial state radiation. For the narrow resonances  $J/\psi$  and  $\psi(2S)$ , the observed cross sections are also distorted by the energy spread of the collider. The radiative corrected cross section is expressed as [12]

$$\sigma_{r.c.}(\sqrt{s}) = \int_{0}^{x_m} dx F(x, s) \frac{\sigma_{Born}(s(1-x))}{|1 - \Pi(s(1-x))|^2} , \qquad (7)$$

where  $\sigma_{Born}$  is the Born order cross section. In the upper limit of the integration  $x_m = 1 - s_m/s$ ,  $\sqrt{s_m}$  is the experimentally required minimum invariant mass of the final state f after losing energy to multi-photon emission; F(x,s) has been calculated in several references [12–14] and  $\Pi(s(1-x))$  is the vacuum polarization factor.

The  $e^+e^-$  colliders have finite energy resolution. The energy resolution function  $G(\sqrt{s}, \sqrt{s'})$  is usually described by a Gaussian distribution:

$$G(\sqrt{s}, \sqrt{s'}) = \frac{1}{\sqrt{2\pi}\Delta} e^{-\frac{(\sqrt{s} - \sqrt{s'})^2}{2\Delta^2}},$$
 (8)

where  $\Delta$ , a function of the beam energy, is the C.M. energy spread of the accelerator. So the experimentally measured resonance cross section,  $\sigma_{exp}$ , is the radiative corrected cross section  $\sigma_{r.c.}$  folded with the energy resolution function:

$$\sigma_{exp}(\sqrt{s}) = \int_{0}^{\infty} \sigma_{r.c.}(\sqrt{s'}) G(\sqrt{s'}, \sqrt{s}) d\sqrt{s'} \quad . \tag{9}$$

Through out this paper, all the physical quantities to be discussed are experimentally observed ones and the radiative correction as well as collider energy spread are implicitly taken into account.

In principle, any experiment measures  $\sigma'_{exp}$  should subtract the contribution of the continuum part to get the physical quantity  $\sigma_{exp}$ , where  $\sigma'_{exp}$  and  $\sigma_{exp}$  indicate the experimental cross sections calculated from Eq.(9) with the substitution of  $\sigma'$  and  $\sigma$  from Eq.(3) and Eq.(1) respectively to  $\sigma_B$  in Eq.(7). Unfortunately, up to now, most of the experiments just neglect this contribution and  $\sigma_{exp} = \sigma'_{exp}$  is assumed for almost all the channels studied, at least at  $J/\psi$  and  $\psi(2S)$ . The noticeable exceptions are  $\Gamma_{ee}$ ,  $\Gamma_{\mu\mu}$  and the total width. These quantities are measured, together with the resonance mass, by scanning around the peak of the resonance and then fitting the measured curves with the theoretical cross sections. In the fitting, the theoretical cross sections always include a continuum term[15].

The difference between  $\sigma_{exp}$  and  $\sigma'_{exp}$  implies a plausible paraphrase in high energy physics literatures. On one hand, the theoretical analyses are based on  $\sigma_{exp}$ , on the other hand, the experiments actually measure  $\sigma'_{exp}$ . However, even in the case that the continuum amplitude is relatively small, such as in  $\psi(2S)$ , certain values of the phase possibly lead to non-negligible interference. For  $\psi(3770)$  scan experiment, the inclusive continuum hadron cross section is larger than the resonance peak, possible interference may contribute a substantial part of the observed cross section.

We now display the effect from the continuum amplitude and corresponding phase for  $J/\psi$ ,  $\psi(2S)$  and  $\psi(3770)$ . To do this, we calculate the ratio

$$k \equiv \frac{\sigma'_{exp} - \sigma_{exp}}{\sigma'_{exp}} \quad . \tag{10}$$

In order to see the effect of the relative phase, the magnitude of  $a_{3g}$ ,  $a_{\gamma}$  and  $a_c$  are treated as input. In principle,  $a_{3g}$ ,  $a_{\gamma}$  and  $a_c$  are different for different exclusive modes both in absolute value and in the relative strength. For illustrative purpose, following assumption is used for any of the exclusive mode: the squared moduli of  $a_{3g}$  and  $a_{\gamma}$  are proportional to their branching ratios of inclusive hadrons  $B(\mathcal{R} \to ggg \to hadron)$  and  $B(\mathcal{R} \to \gamma^* \to hadron)$  given by PDG [16], and the squared modulus of  $a_c$  is assumed to be proportional to the Born order  $\mu^+\mu^-$  cross section multiplied by  $R_{had}$ 

TABLE I: Amplitude estimation for three charmonium states [16].  $\sigma_{\mathcal{R}}$  is total cross section for resonance  $\mathcal{R}$ , ( $\mathcal{R} = J/\psi, \psi(2S)$ , and  $\psi(3770)$ ).

	$J/\psi$	$\psi(2S)$	$\psi(3770)$
$ a_{3g} ^2$	$60\% \ \sigma_{J/\psi}$	$15\% \ \sigma_{\psi(2S)}$	
$ a_{\gamma} ^2$	$17\% \ \sigma_{J/\psi}$	$2.9\% \ \sigma_{\psi(2S)}$	$3 \times 10^{-5} \sigma_{\psi(3770)}$
$ a_c ^2$	20 nb	15 nb	13 nb

which indicates the hadronic cross section of the continuum process, and is estimated by pQCD [16]. Table I lists these inputs for  $J/\psi$ ,  $\psi(2S)$  and  $\psi(3770)$ .

If we use the BEPC energy spread listed by PDG [16],  $\sigma_{J/\psi} \simeq 3100$  nb,  $\sigma_{\psi(2S)} \simeq 700$  nb, and  $\sigma_{\psi(3770)} \simeq 8$  nb are got. Combining with equation (1), (3) and (10), we could obtain k as a function of  $\phi$  and  $\phi'$ , whose variation is shown in Fig. 2. It can be seen that for certain values of the two phases, k could deviates from 0, or equivalently the ratio  $\sigma'_{exp}/\sigma_{exp}$  deviates from 1, which implies that the continuum process may produce non-negligible effect in experimental measurement.

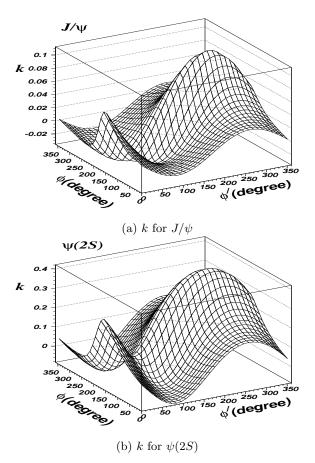


FIG. 2: k as a function of  $\phi$  and  $\phi'$ , (a) for  $J/\psi$  and (b) for  $\psi(2S)$ .

#### III. $a_c$ IN $e^+e^- \rightarrow c\bar{c}$ PROCESSES

### **A.** $\psi(3770)$

The  $\psi(3770)$  is a wide resonance with  $\Gamma_{tot} = (23.6 \pm$ 2.5) MeV [16]. The collider energy spread can be neglected as long as the standard deviation of the energy resolution function  $\Delta$  is comparatively small, such as  $\Delta \leq 2$  MeV. Its maximum cross section of inclusive hadrons in  $e^+e^-$  collision is 8 nb, while the continuum cross section is about 13 nb. The resonance predominantly decays into  $D\overline{D}$ , while the continuum cross section mainly goes to light hadrons. The decay rate of  $\psi(3770)$  to light hadrons via three-gluon annihilation even though small in the total width, e.g. at the order of 1%, or partial width of about 230 keV, which means that  $|a_{3q}| \approx 0.08 |a_c|$  ( $\psi(3770)$  decays to light hadrons via one-photon annihilation is three orders of magnitude lower than the three-gluon annihilation given above [16], and could be neglected), if it has interference with continuum amplitude, it could bring an interference of maximum 1.9 nb in the observed cross section to light hadrons. Another interference between the tail of  $\psi(2S) \to a_{3g}$  and  $a_c$ at  $\sqrt{s} = m_{\psi(3770)}$  brings an interference of maximum 1 nb, while the interference between  $\psi(2S)$  and  $\psi(3770)$  at  $\sqrt{s} = m_{\psi(3770)}$  is small, with 0.07 nb at maximum. The continuum cross section of  $e^+e^- \to D\overline{D}$ , in the simple naive quark model, is estimated by

$$\sigma_c(s) = \frac{4}{3} \cdot \sigma_{\mu^+\mu^-}(s) \cdot \frac{2P_D}{E_{cm}},\tag{11}$$

with  $P_D$  being the momentum of the D or  $\overline{D}$ . After taken into account of radiative correction, it is 0.019nb. But the interference between  $a_c$  and  $a_D$  is 0.79nb at maximum. Here  $a_D$  denotes the OZI allowed strong interaction amplitude which is responsible for  $\psi(3770) \to D\overline{D}$  decays.

A possible large constructive interference for light hadrons and, at the same time, a large destructive interference for  $D\overline{D}$ , could be responsible for the larger cross section of inclusive hadrons by direct measurement of  $e^+e^- \to \psi(3770) \to hadrons$  [17] than the  $D\overline{D}$  cross section measured by Mark III Collaboration using D singletag and double-tag [18].

As to the exclusive decays, it could make some of the decay modes with small branching ratios more observable at the resonance. For example, if the missing decay modes of  $\psi(2S)$  like  $\rho\pi$  do appear in  $\psi(3770)$  decays, with an enhancement factor [19], their on-resonance cross section could be substantially larger than off-resonance in  $e^+e^-$  experiment. Quantitatively, if  $\mathcal{B}(\psi(3770) \to \rho\pi) \approx 4 \times 10^{-4}$  (or equivalently,  $\sigma_{\psi(3770) \to \rho\pi} \approx 0.003$  nb) as suggested in Ref. [19], and  $\sigma(e^+e^- \to \rho\pi) \approx 0.014$  nb at Born order by the model of Ref. [20]\*, then the maximum interference could be 0.011 nb, much larger than

the pure contribution from  $\psi(3770) \to \rho \pi$ . Comparing the cross sections on and off  $\psi(3770)$  peak,  $\psi(3770) \to \rho \pi$  could be seen through the interference with the continuum amplitude.

**B.** 
$$\psi(2S)$$

As can be seen in Fig. 2 (b), the ratio  $\sigma'_{exp}/\sigma_{exp}$  could deviate from 1 substantially. For each exclusive decay channels, k could be different, due to the magnitudes of  $a_{\gamma}$  and  $a_{3g}$ , have different coefficients and  $a_c$ , if estimated by form factors, are of different functions of the energy. This must be taken into account in the fitting of  $a_{\gamma}$ ,  $a_{3g}$  and the phase in between. In general, with the interference between  $a_c$  and the resonance, the maximum height of each exclusive channel does not necessarily coincide with the maximum height of the inclusive hadrons on which data are taken. We shall take the  $\mu^+\mu^-$  channel as an example in the next section.

In  $\psi(2S)$  final state analyses, it is noticeable that the observed cross sections of some electromagnetic processes, such as  $\psi(2S) \to \pi^+\pi^-$ ,  $\psi(2S) \to \omega\pi^0$ , and the famous puzzling process  $\psi(2S) \to \rho \pi$ , are three to four orders of magnitude smaller than the inclusive hadron cross section of the continuum process, which is about 15 nb. Form factor estimation [21] gives these cross sections comparable to the magnitudes off the resonance [22]. It implies that a substantial part of the experimentally measured cross section could comes from the continuum amplitude  $a_c$  instead of the  $\psi(2S)$  decays, and interference between these two amplitudes may even affect the measured quantities further. Therefore it is essential to know the production rate of  $\pi^+\pi^-$ ,  $\omega\pi^0$  and  $\rho\pi$  due to the continuum process in order to get their correct branching ratios of the  $\psi(2S)$  decays.

In order to know whether the observed suppression of VP and VT modes in  $\psi(2S)$  decays are due to the absence of strong interaction amplitude, or the destructive interference between the electromagnetic and the strong amplitudes, or just an incidental destructive interference between these two and the continuum process in particular experiment, the amplitude  $a_c$  must be taken into account.

C. 
$$J/\psi$$

From Fig. 2 (a), it is seen that the interference between the amplitude  $a_c$  and the resonance is at the order of a few percent which is much smaller than that of  $\psi(2S)$ .

<sup>[\*]</sup> The same calculation gives  $\rho\pi$  cross section at the mass of  $\psi(2S)$ 

to be 0.015 nb, which is just below the current upper limit of the branching ratio  $2.8 \times 10^{-5}$  or upper limit of the cross section 0.02 nb by BES [6].

It is also smaller than the statistical and systematic uncertainties of current measurements. Nevertheless, for future high precision measurements such as the proposed CLEO-c [1] and BES-III [2], when the accuracy goes to a few per mille level, it should be taken into account.

# IV. THE DEPENDENCE ON EXPERIMENTAL CONDITIONS

In this section, we discuss the dependence of the observed cross section in  $e^+e^-$  collision on the experimental conditions. The most crucial experiment conditions are the accelerator energy spread and the beam energy setting. The former will smear the intrinsic width of the resonance so that change the relative contribution between the resonance and the continuum, while the latter will affect the relative contribution as well as the absolute correction to the total rate due to the interference. The invariant mass cut or equivalent requirement in data analysis will also affect the relative contribution of resonance and continuum due to the different energy dependence of the cross sections in radiative correction.

#### A. Dependence on collider energy resolution

For narrow resonance like  $J/\psi$  and  $\psi(2S)$ , with intrinsic widths much narrower than the energy resolution of the current  $e^+e^-$  colliders, we don't observe their original resonance curve. Instead, what we actually measure is the resonance smeared by the finite energy resolution of the collider. In Fig. 3, three cross sections are depicted: the Breit-Wigner cross section, the cross section after radiative correction (Eq.(7)), and the experimentally measured cross section (Eq.(9)).

In actual experiments, data are naturally taken at the energy which yields the maximum height of the inclusive hadrons. This energy is not the nominal mass of the resonance but somewhat higher, neither does it coincide with the maximum height of each exclusive channels due to the interference effect with  $a_c$ . For comparison, Fig. 4, depicts the observed cross sections of inclusive hadrons and  $\mu^+\mu^-$  pairs. Two arrows in the figure denote the different positions of the maximum heights of the cross sections. It is well know that the radiative correction reduces the height of the peak and shift the maximum height of the resonance peak upwards, and the energy resolution of the collider both reduces the height of the peak and shifts it more profoundly. Such shift, depends on the energy resolution of the collider, in general could be different for inclusive hadrons and for each exclusive channels. For example, the peak of  $\mu^+\mu^-$  curve is shifted more than that of the inclusive hadrons, to 0.81 MeV above the  $\psi(2S)$  nominal mass for BEPC energy spread, which is 1.3 MeV.

It is clear that the shape of the observed cross section is much different from that of Breit-Wigner. However, the

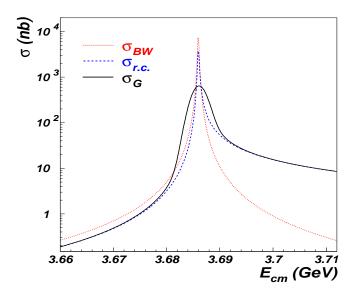


FIG. 3: Total cross section at  $\psi(2S)$ :  $\sigma_{BW}$  for Breit-Wigner cross section,  $\sigma_{r.c.}$  the cross section with radiative correction, and  $\sigma_{exp}$  the measured cross section on a collider with  $\Delta=1.3$  MeV.

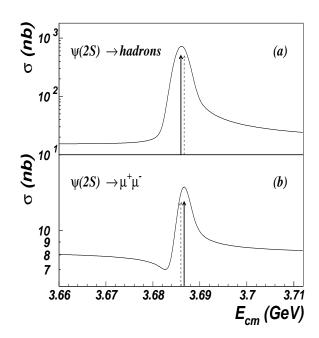


FIG. 4: Cross sections in the vicinity of  $\psi(2S)$  for hadron (a) and  $\mu^+\mu^-$  (b) final states. The solid line with arrow indicates the peak position and the dashed line with arrow the shift of the other peak position.

energy smear hardly affects the continuum part of the cross section. So in the observed cross section, what proportion comes from the contribution of continuum and interference is sensitive to the energy spread. The larger the collider energy spread is, the more share the continuum part contributes in the observed cross section.

#### B. Dependence on beam energy

For demonstration of such dependence, we show the curve of  $\mu^+\mu^-$  channel, for its dynamics is clear and there is no unknown parameter. It is similar to those hadronic channels in  $\psi(2S)$  decays which only go through electromagnetic interaction, such as  $\omega\pi^0$  and  $\pi^+\pi^-$ . Since this is an exclusive channel, there is interference between the continuum and the  $\psi(2S)$  amplitudes. Such interference can be seen clearly from the scan of the  $\psi(2S)$ , as shown in Fig. 5.

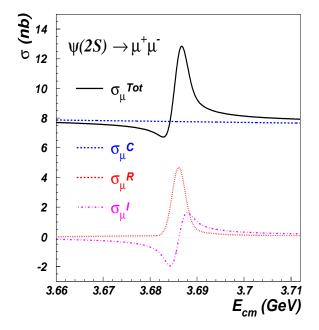


FIG. 5: Contributions of three parts to the cross section: dashed line for QED continuum  $(\sigma^C)$ ; dotted line for resonance  $(\sigma^R)$ ; dash dotted line for interference  $(\sigma^I)$ ; solid line for total cross section  $(\sigma^{Tot})$ .

The relative contribution of the resonance and the continuum changes rapidly as the energy changes, and the interference term between these two amplitudes also varies with energy. The latter could change from negative to positive as the energy passes across the nominal mass of the resonance. In the actual experimental situation, it is important to know the beam energy precisely, which is needed in the delicate task to subtract the contribution of  $a_c$ . Possible uncertainty and drift of the beam energy need to be taken into account in the determination of the systematic errors.

#### C. Dependence on invariant mass requirement

The magnitude of the continuum part of the cross section is sensitive to the upper limit of the integration in the calculation of the radiative correction, i.e., it depends sensitively on the invariant mass cut in the experiment (Eq. (7)). This is because that the Born order cross

section of the continuum term goes up as C.M. energy decreases. If the event selection uses a very loose cut, then the cross section from continuum part could be very large. This is particularly true for exclusive channels, because QCD predicts the form factors to be powers of  $1/Q^2$ . While the resonance part is not sensitive to the invariant mass cut, since the Breit-Wigner formula serves as a natural cut by itself. Mathematically, in the upper limit of the integration of Eq. (7), as long as  $(1 - s_m/s) \gg \Gamma/M$ , the integration is not sensitive to the upper limit, where  $\Gamma$  and M are the total width and mass of the resonance respectively.

So in the observed cross section, what proportion comes from the contribution of continuum and interference is sensitive to the events selection criteria. Many of a time, it is not the invariant mass cut directly applied to the data, instead it is affected by many cuts, like the momentum cut, kinematic fit, collinearity cut and so on. In this case, how much is the contribution of the continuum and the interference could only be calculated by Monte Carlo simulation. Qualitatively, looser invariant mass requirement in event selection would increase the share of the continuum part of the contribution.

It is worth noting here that in principle if  $a_c$  is not considered correctly, different experiments will give different results to the same quantity, like the exclusive branching ratio of the resonance, due to the dependence on beam energy spread, beam energy setting, and invariant mass requirement in event selection. This point is especially important for the time being, since the beam spreads for different accelerators are much different, and events selection criteria is very different because of the big background in the channels analyzed [23].

#### V. SUMMARY AND PERSPECTIVES

As we have emphasized in the foregoing discussion, the amplitude  $a_c$ , by itself or through interference with the resonance, could contribute significantly to the observed cross sections in  $e^+e^-$  experiments on charmonium physics. Its treatment depends sensitively on the experimental details, this has not been fully addressed in both  $e^+e^-$  experiments and theoretical analyses based on these results. So far, most of the measurements are crude, with large statistical and systematic uncertainties, so this problem has been outside of the purview of concern. Now with large  $J/\psi$  and  $\psi(2S)$  samples from BES-II [24] and forthcoming high precision experiments CLEO-c [1] and BES-III [2], the effect of  $a_c$  needs to be addressed properly.

To study the continuum contribution, the most promising way is to do energy scan for every exclusive mode in the vicinity of the resonance, so that both the amplitudes and the relative phases could be fit out simultaneously. In case this is not practicable, data sample off the resonance with comparable integrated luminosity as on

the resonance should be collected to measure  $|a_c|$ , which could give an estimation of its contribution in the decay modes studied. The theoretical analyses based on current available  $e^+e^-$  data, particularly on  $\psi(2S)$  may need to be revised correspondingly.

In fact, another way to free ourselves from the effect of the continuum is to analyze the decay product from higher energy experiments. For example,  $J/\psi$  decays could be measured from  $\psi(2S) \to \pi^+\pi^- J/\psi$ ,  $\psi(2S)$  could be studied from  $p\bar{p}$  annihilation experiments or from B decays at the B factories.

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